Lecture 25

TM accepting a language

TM accepting a language

- Definition

  Let $T=(Q, \Sigma, \Gamma, \delta, s)$ be a TM, and $w \in \Sigma^*$.

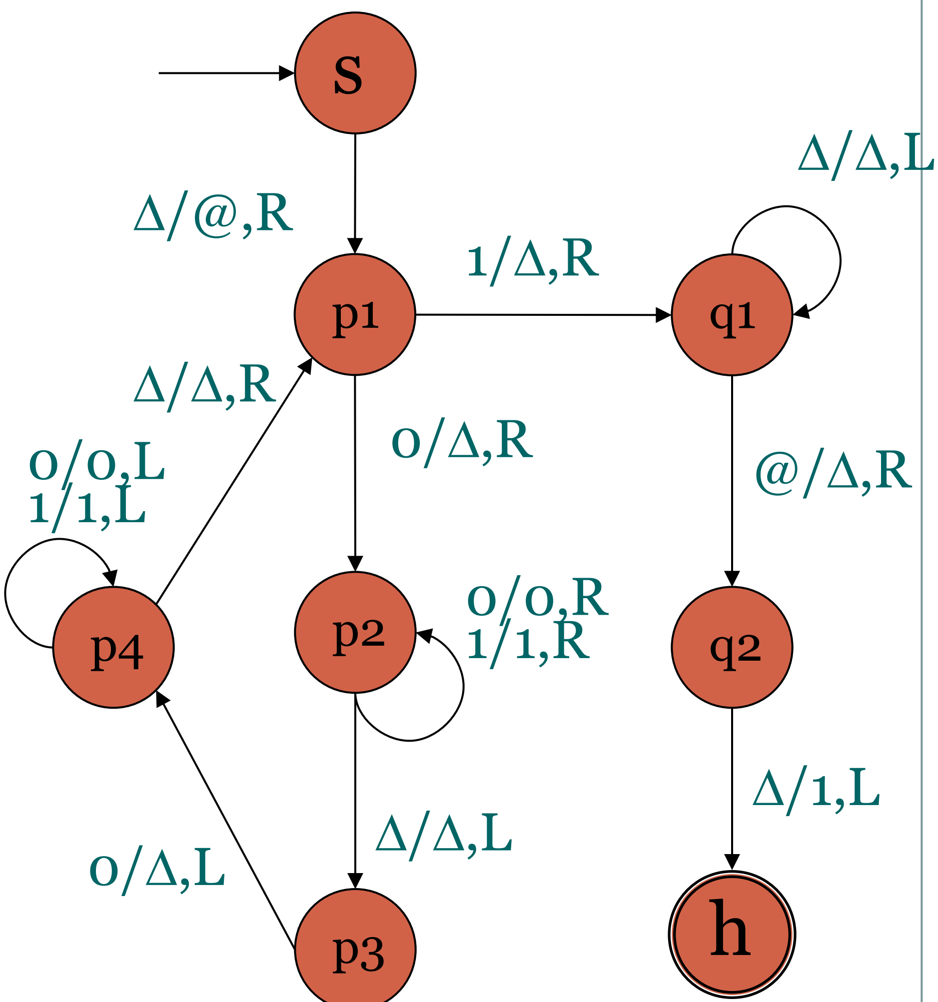
  T accepts w if $(s, \varepsilon, \Delta, w) \vdash_T^* (h, \varepsilon, \Delta, 1)$.

  The language accepted by a TM T, denoted by $L(T)$, is the set of strings accepted by T.

Example of language accepted by a TM

$L(T)=\{0^n10^n \mid n\geq 0\}$

- T halts on 0^n10^n
- T hangs on $0^{n+1}10^n$ at p3
- T hangs on 0^n10^{n+1} at q1
- T hangs on $0^n\ 1^2\ 0^n$ at q1



TM computing a function

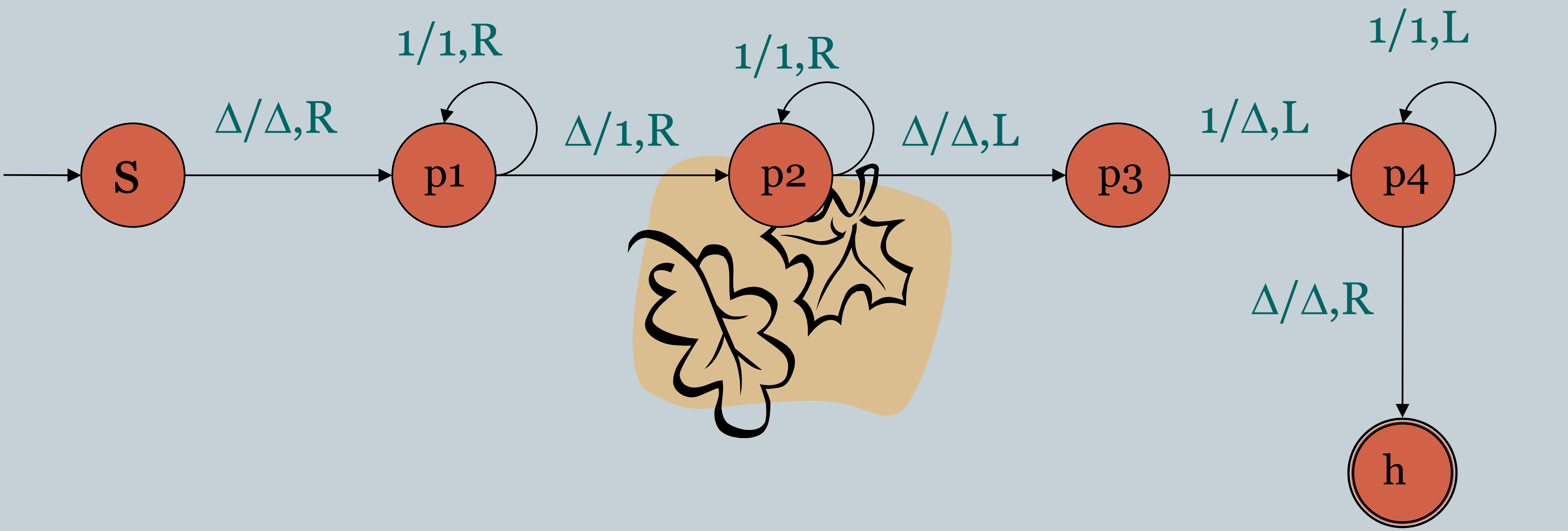
- Definition

  Let $T=(Q, \Sigma, \Gamma, \delta, s)$ be a TM, and f be a function from Σ^* to Γ^*.

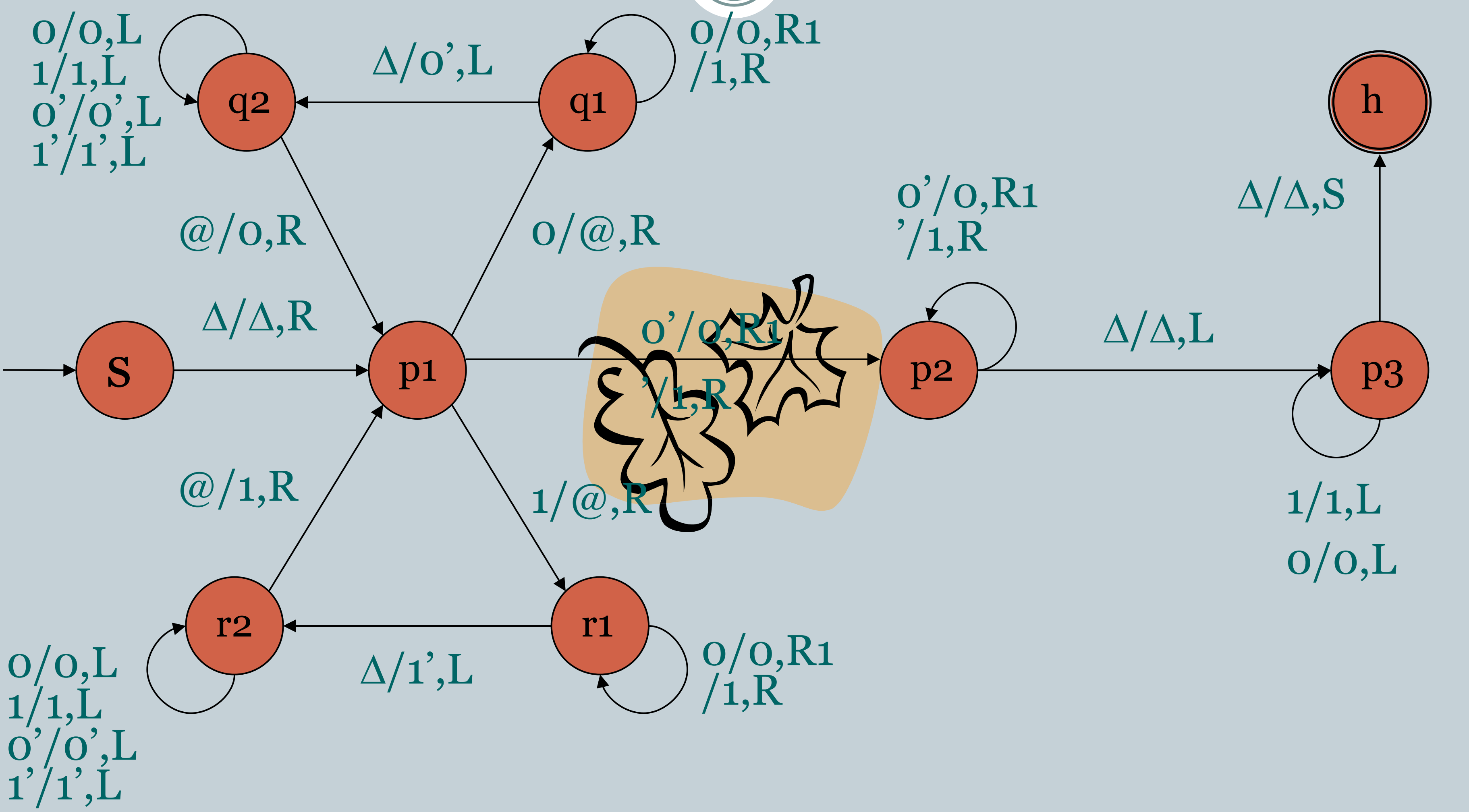
  T computes f if, for any string w in Σ^*,

  $(s, \varepsilon, \Delta, w) \vdash_T^* (h, \varepsilon, \Delta, f(w))$.

Example of TM Computing Function



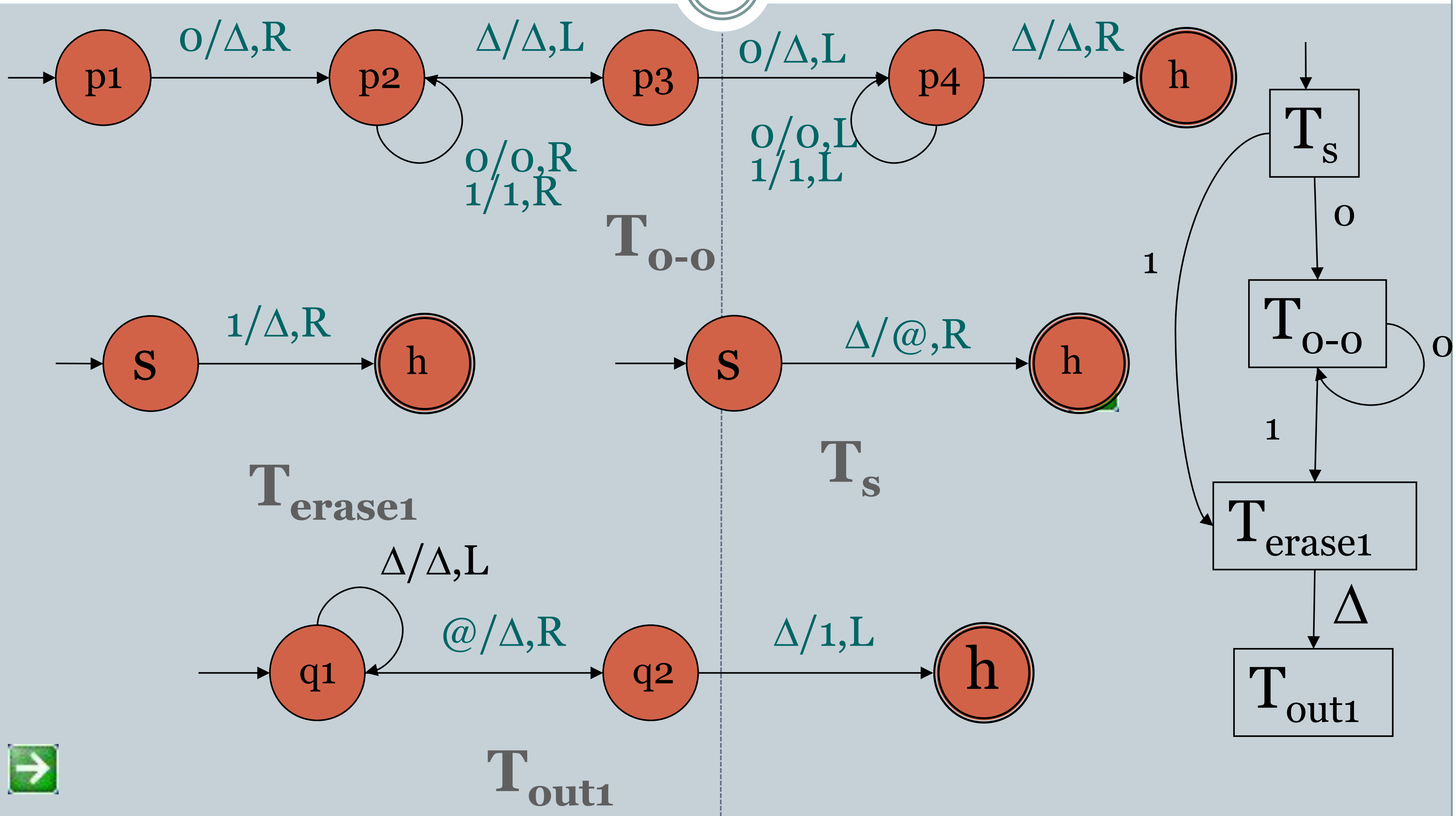
Example of TM Computing Function



Composite TM

- Let *T1* and *T2* be TM's.
- *T1* $\rightarrow$ *T2* means executing *T1* until *T1* halts and then executing *T2*.
- *T1* $\xrightarrow{a}$ *T2* means executing *T1* until *T1* halts and if the symbol under the tape head when *T1* halts is a then executing *T2*.

Example of Composite TM



Nondeterministic TM

- An NTM starts working and stops working in the same way as a DTM.
- Each move of an NTM can be nondeterministic.

Each Move in an NTM

- reads the symbol under its tape head
- According to the *transition relation* on the symbol read from the tape and its current state, the TM choose one move nondeterministically to:
  - write a symbol on the tape
  - move its tape head to the left or right one cell or not
  - changes its state to the *next state*

How to define nondeterministic TM (NTM)

- a quintuple $(Q, \Sigma, \Gamma, \delta, s)$, where
  - the set of states Q is finite, and does not contain halt state h,
  - the input alphabet Σ is a finite set of symbols, not including the blank symbol Δ,
  - the tape alphabet Γ is a finite set of symbols containing Σ, but not including the blank symbol Δ,
  - the start state s is in Q, and
  - the transition fⁿ $\delta: Q\times(\Gamma\cup\{\Delta\})\rightarrow 2^{Q\cup\{h\}\times(\Gamma\cup\{\Delta\})\times\{L,R,S\}}$.

Configuration of an NTM

Definition

- Let $T = (Q, \Sigma, \Gamma, \delta, s)$ be an TM.

  A configuration of T is an element of $Q \times \Gamma^* \times \Gamma \times \Gamma^*$
- Can be written as
  - (q, l, a, r) or
  - $(q, l \cdot \underline{a} \cdot r)$

string to the left of tape head

symbol under tape head

string to the right of tape head

Yield the next configuration

Definition

- Let $T = (Q, \Sigma, \Gamma, \delta, s)$ be an NTM, and $(q_1, \alpha_1\underline{a_1}\beta_1)$ and $(q_2, \alpha_2\underline{a_2}\beta_2)$ be two configurations of T.

  We say $(q_1, \alpha_1\underline{a_1}\beta_1)$ yields $(q_2, \alpha_2\underline{a_2}\beta_2)$ in one step, denoted by $(q_1, \alpha_1\underline{a_1}\beta_1) \vdash_T (q_2, \alpha_2\underline{a_2}\beta_2)$, if
  - $(q_2, a_2, S) \in \delta(q_1, a_1)$, $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$,
  - $(q_2, b, R) \in \delta(q_1, a_1)$, $\alpha_2 = \alpha_1 b$ and $\beta_1 = a_2\beta_2$,
  - $(q_2, b, L) \in \delta(q_1, a_1)$, $\alpha_1 = \alpha_2 a_2$ and $\beta_2 = b\beta_1$.

NTM accepting a language/computing a function

- Definition

Let $T = (Q, \Sigma, \Gamma, \delta, s)$ be an NTM.

Let $w \in \Sigma^*$ and f be a function from Σ^* to Γ^*.

T accepts w if $(s, \varepsilon, \Delta, w) \vdash_T^* (h, \varepsilon, \Delta, 1)$.

The language accepted by a TM T, denoted by $L(T)$, is the set of strings accepted by T.

T computes f if, for any string w in Σ^*, $(s, \varepsilon, \Delta, w) \vdash_T^* (h, \varepsilon, \Delta, f(w))$.

Example of NTM

- Let $L = \{ww \mid w \in \{0,1\}^*\}$